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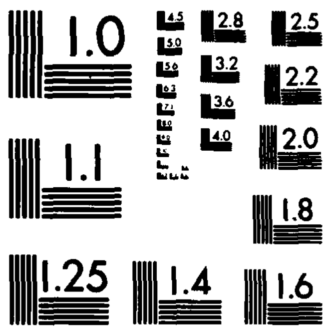

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ABSTRACT

This paper examines the problem of target localization based solely upon bearing information obtained by a single platform over short observation times. A number of Monte Carlo and analytical techniques for the construction of statistical distributions of target ranges are presented and compared.

1. INTRODUCTION

If an observer travels in a straight path at a known constant velocity and observes a stationary target at two different exact bearings separated by time t, the range to the target is given by

$$R_{predicted} = V_A \cdot t \frac{\sin \phi_0}{\sin (\phi_1 - \phi_0)} \tag{1.1}$$

Here V_A is the velocity of the observer and ϕ_1 and ϕ_0 are the final and initial observed bearings, respectively. If, on the other hand, the target moves with velocity V_s and each observed bearing can differ from the actual bearing to the target by as much as $\pm\delta$, the range to the target could be as great as

$$R_{max} = \frac{[V_A \sin(\phi_0 + \delta) + V_S] \quad t}{\sin(\phi_1 - \phi_0 - 2\delta)} \quad , \tag{1.2}$$

or as little as

$$R_{min} = \frac{[V_A \sin(\phi_0 - \delta) - V_S] \quad t}{\sin (\phi_1 - \phi_0 + 2\delta)} \quad . \tag{1.3}$$

(see figure 1).

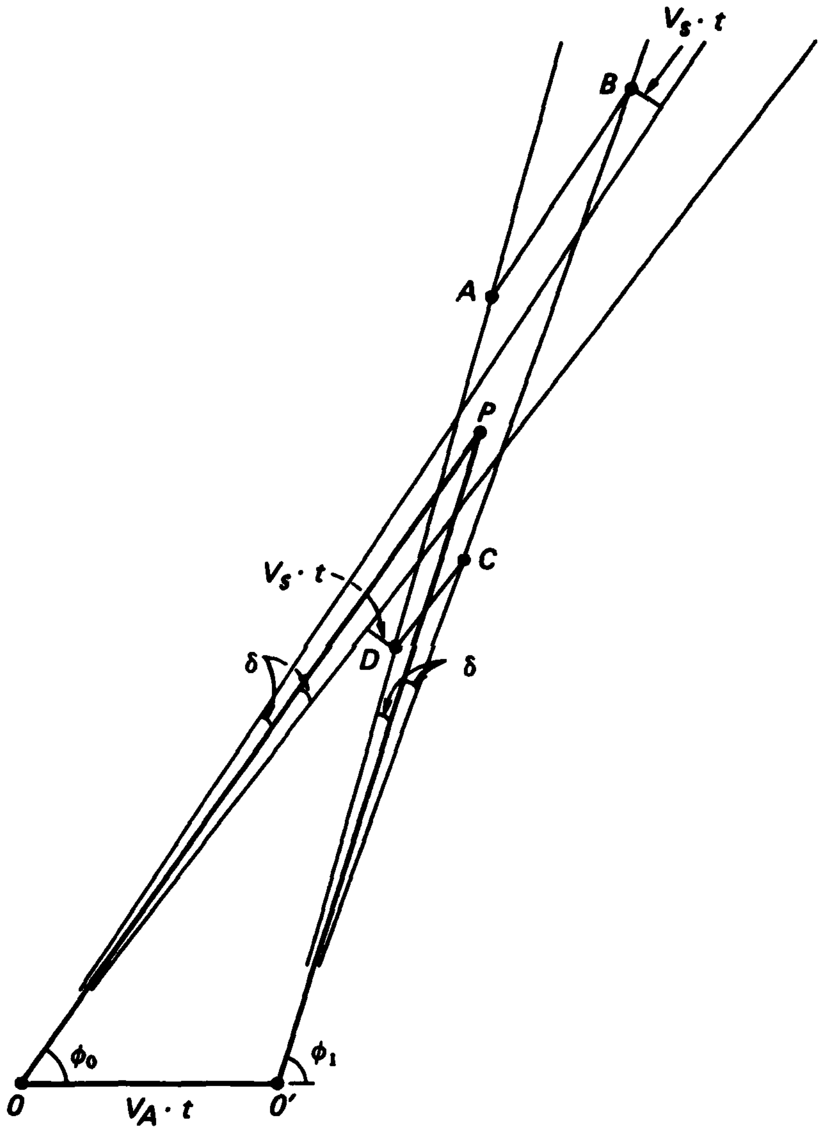

FIG. 1: GEOMETRY OF TWO-BEARING TRIANGULATION

The actual range to the target depends upon the target motion and the actual bearing errors; if the bearings are noisy, an appropriate question is: what is the distribution of range, given the observed bearings?

In this paper, a number of Monte Carlo and analytical techniques for the construction of the range distribution are given. Sections 2 and 3 contain descriptions and results of the Monte Carlo simulations. Sections 4 and 5 contain analytical results. In section 2 a "random range" simulation is described. In section 3, a "random bearing" simulation is described. In section 4, a method for approximately calculating the mean and variance of the range, given the observations, is presented. In section 5, a method for approximately calculating the distribution on the range is presented. Section 5 contains a discussion of the tactical use of the results presented in sections 2 to 4.

2. THE RANDOM RANGE SIMULATION

Description of the Simulation

This simulation runs for specified values of the observed initial and final bearings. The actual final bearings are assumed to be uniformly distributed between $+\delta$ and $-\delta$ of the observed final bearings. The actual initial bearings are assumed to be within $\pm\delta$ of the observed initial bearings with no assumption as to the distribution. Final target ranges are assumed to be uniformly distributed between R_{min} and the lesser of R_{max} or a specified range bound (hence, the name random range simulation.) Target motion is assumed to be in a uniformly random direction at a constant velocity, V_s.

The simulation begins with the final observed bearing. It selects a bearing error to determine the actual final bearing, then selects a final target range, R. It selects the direction of target motion, then traces motion of target and observer backwards in time to their positions when the initial bearing was observed. If the actual bearing to the target at this point is within $+\delta$ or $-\delta$ of the observed initial bearing, R is retained as a valid sample. Otherwise, it is discarded. The procedure is repeated until the desired sample size has been obtained.

Simulation Results

Assume that the observer is an aircraft and the target is a ship. The aircraft ground velocity was 420 kts, the target velocity 30 kts, and the target range was not allowed to exceed 75 nautical miles. The time between bearing observations ranged from 30 to 120 seconds in 15 second increments. The initial observed bearing was 30 degrees in all instances, with bearing changes ranging from 2 to 10 degrees. Final bearing errors were assumed to be uniformly distributed up to 1 degree either side of the observed bearing. One thousand replications were collected for each combination of bearing change and time between bearings. Results of these simulations are shown in and figures 2 through 8.

3. RANDOM BEARING SIMULATION

Description of the Simulation

The simulation described in this section works directly with the distribution on R, instead of assuming a distribution and then rejecting trials in which the assumptions are not met. Assume that the bearing changes are ϕ_0 and ϕ_1, that the aircraft and ship speeds are V_A and V_S, that the observation time is t, and that the target moves in the direction α. In the next section it is shown that the range R to the target after the second bearing is

$$R = \frac{V_A t \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - \frac{V_S t \sin(\phi_0 - \alpha)}{\sin(\phi_1 - \phi_0)} \qquad (3.1)$$

The simulation proceeds as follows:

1. Initialize, N=0. Input observed bearings $\hat{\phi}_0$, $\hat{\phi}_1$.

2. Select α, where α is uniformly distributed on [0, 360°].

TABLE 1

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 30 seconds R < 75 n.mi.

R (n.mi.)	Bearing change (degrees) 2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
5									
10						1.0	1.0	1.0	1.0
15		1.0	1.0	1.0	1.0	.986	.969	.896	.801
20	1.0	.998	.996	.965	.907	.820	.669	.450	.232
25	.989	.979	.938	.828	.682	.487	.260	.127	.036
30	.946	.903	.825	.636	.434	.215	.058	.012	0
35	.874	.792	.677	.439	.235	.065	.008	0	
40	.779	.672	.519	.293	.100	.016	0		
45	.669	.528	.416	.184	.049	.001			
50	.546	.429	.309	.128	.010	0			
55	.413	.329	.213	.070	0				
60	.301	.227	.131	.036					
65	.203	.147	.076	.011					
70	.103	.075	.027	.003					

*1,000 replications per bearing change with ϕ_0 = 30°.

TABLE 2

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 45 seconds R < 75 n.mi.

R	Bearing change (degrees)								
(n.mi.)	3.0	3.5	4.0	4.5	5.0	5.5	6.0	7.0	8.0
5									
10								1.0	1.0
15				1.0	1.0	1.0	1.0	.996	.951
20		1.0	1.0	.999	.998	.979	.936	.752	.442
25	1.0	.993	.987	.948	.890	.762	.623	.287	.055
30	.987	.958	.894	.799	.651	.422	.279	.037	0
35	.934	.883	.734	.555	.384	.203	.078	0	
40	.846	.756	.558	.360	.172	.055	.013		
45	.727	.592	.395	.205	.066	0	0		
50	.592	.444	.266	.097	.016				
55	.431	.296	.171	.047	.003				
60	.314	.191	.089	.016	0				
65	.174	.115	.044	.001					
70	.088	.056	.019	0					

*1,000 replications per bearing change with ϕ_0 = 30°.

TABLE 3

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 60 seconds R < 75 n.mi.

R	Bearing change (degrees)								
(n.mi.)	4.0	4.5	5.0	5.5	6.0	7.5	8.0	9.5	10.0
5									
10									1.0
15						1.0	1.0	1.0	.993
20			1.0	1.0	1.0	.992	.951	.813	.582
25	1.0	1.0	.999	.992	.676	.853	.577	.284	.097
30	.997	.991	.953	.902	.798	.521	.186	.027	0
35	.960	.933	.823	.662	.492	.199	.026	0	
40	.895	.814	.630	.413	.257	.041	0		
45	.762	.649	.418	.207	.105	.002			
50	.610	.475	.260	.103	.035	0			
55	.455	.315	.140	.045	.006				
60	.309	.186	.071	.009	0				
65	.187	.094	.028	0					
70	.077	.036	.008						

*1,000 replications per bearing change with $\phi_0 = 30°$.

TABLE 4

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 75 seconds R < 75 n.mi.

R	Bearing change (degrees)								
(n.mi.)	4.0	4.5	5.0	5.5	6.0	7.0	8.0	9.0	10.0
5									
10									
15								1.0	1.0
20						1.0	1.0	.999	.949
25				1.0	1.0	.997	.961	.828	.578
30		1.0	1.0	.998	.982	.903	.689	.424	.125
35	1.0	.996	.985	.942	.904	.642	.336	.102	.002
40	.990	.969	.923	.825	.713	.335	.091	.004	0
45	.947	.885	.799	.652	.480	.124	.014	0	
50	.845	.735	.621	.447	.262	.039	0		
55	.699	.568	.427	.270	.132	.002			
60	.531	.381	.274	.149	.059	0			
65	.364	.225	.154	.066	.022				
70	.181	.090	.056	.029	.002				

*1,000 replications per bearing change with ϕ_0 = 30°.

TABLE 5

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 90 seconds R < 75 n.mi.

R	Bearing change (degrees)									
(n.mi.)	5.0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5										
10										
15										
20							1.0	1.0	1.0	1.0
25				1.0	1.0	1.0	.999	.990	.960	.935
30		1.0	1.0	.996	.973	.946	.894	.796	.692	.572
35	1.0	.988	.968	.930	.842	.728	.598	.440	.307	.203
40	.996	.919	.870	.746	.608	.423	.273	.153	.075	.028
45	.953	.778	.659	.490	.333	.168	076	.023	.005	0
50	.858	.591	.443	.259	.146	.052	.007	0	0	
55	.701	.386	.241	.109	.007	0				
60	.516	.227	.110	.046	010	0				
65	.304	.114	.043	.007	0					
70	.152	.051	.014	0						

*1,000 replications per bearing change with $\phi_0 = 30°$.

TABLE 6

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

V_A = 420 kts V_S = 30 kts t = 105 seconds $R < 75$ n.mi.

R	Bearing change (degrees)									
(n.mi.)	5.0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5										
10										
15										
20										
25							1.0	1.0	1.0	1.0
30			1.0	1.0	1.0	1.0	.995	.980	.937	.892
35		1.0	.999	.998	.983	.957	.875	.822	.686	.630
40	1.0	.993	.975	.945	.892	.796	.633	.515	.350	.283
45	.997	.949	.897	.801	.692	.538	.382	.220	.110	.070
50	.956	.826	.728	.617	.461	.294	.150	.055	.012	.000
55	.859	.651	.532	.382	.251	.109	.042	.007	0	0
60	.710	.444	.340	.214	.128	.033	.004	0		
65	.488	.258	.180	.095	.037	.004	0			
70	.234	.106	.066	.030	.004	0				

*1,000 replications per bearing change with $\phi_0 = 30°$.

TABLE 7

FRACTION OF TRIALS WITH RANGE GREATER THAN R*

$V_A = 420$ kts $V_S = 30$ kts $t = 120$ seconds $R < 75$ n.mi.

R	Bearing change (degrees)									
(n.mi.)	5.0	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5										
10										
15										
20										
25									1.0	1.0
30							1.0	1.0	.998	.991
35				1.0	1.0	1.0	.988	.965	.935	.945
40		1.0	1.0	.979	.977	.953	.896	.825	.717	.565
45	1.0	.997	.979	.954	.879	.794	.696	.560	.422	.252
50	.994	.858	.912	.836	.694	.582	.435	.300	.184	.066
55	.955	.881	.778	.659	.469	.355	.216	.100	.051	.009
60	.857	.707	.580	.442	.276	.187	.072	.020	.003	0
65	.684	.486	.357	.260	.137	.082	.018	.002	0	
70	.372	.236	.163	.097	.042	.024	0	0		

*1,000 replications per bearing change with $\phi_0 = 30°$.

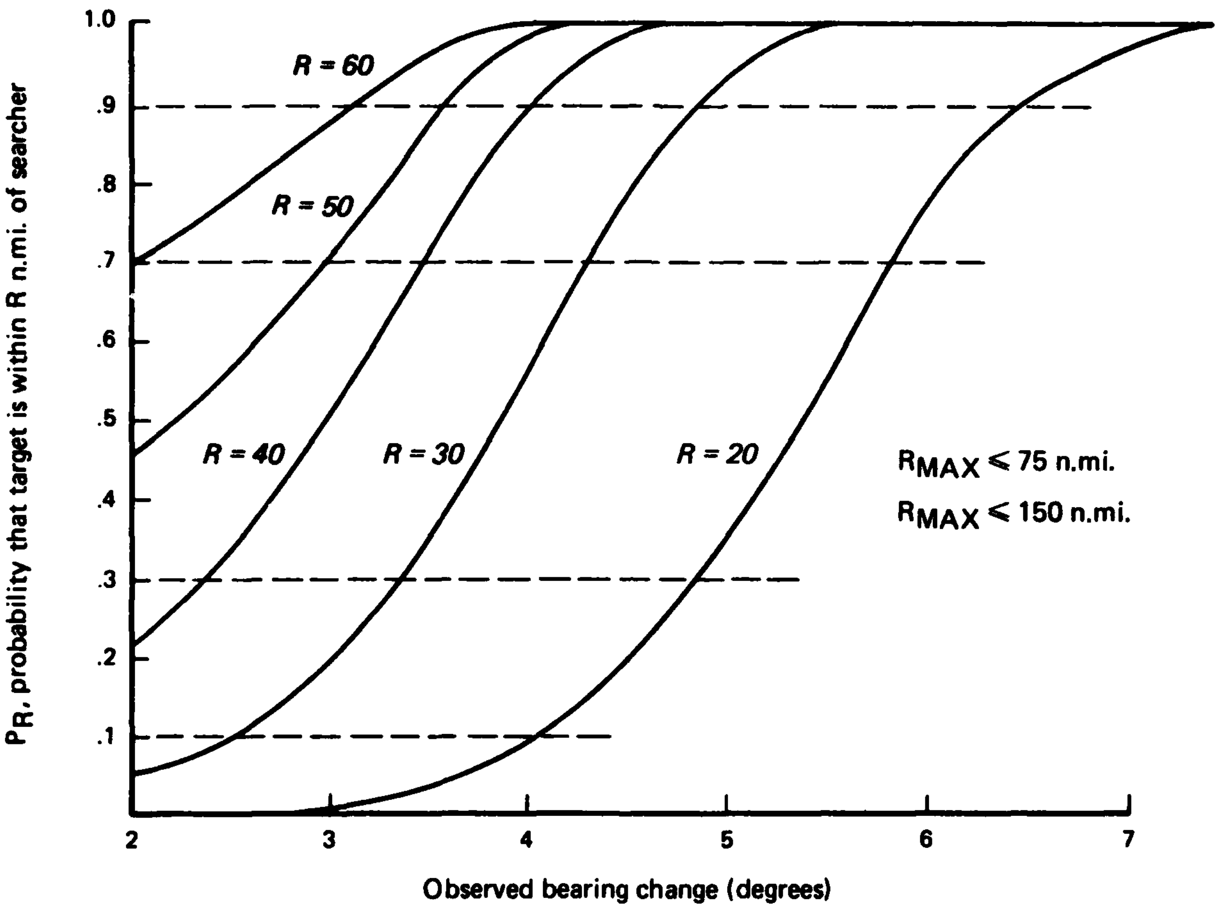

FIG. 2: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 30 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND $R_{MAX} <$ 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

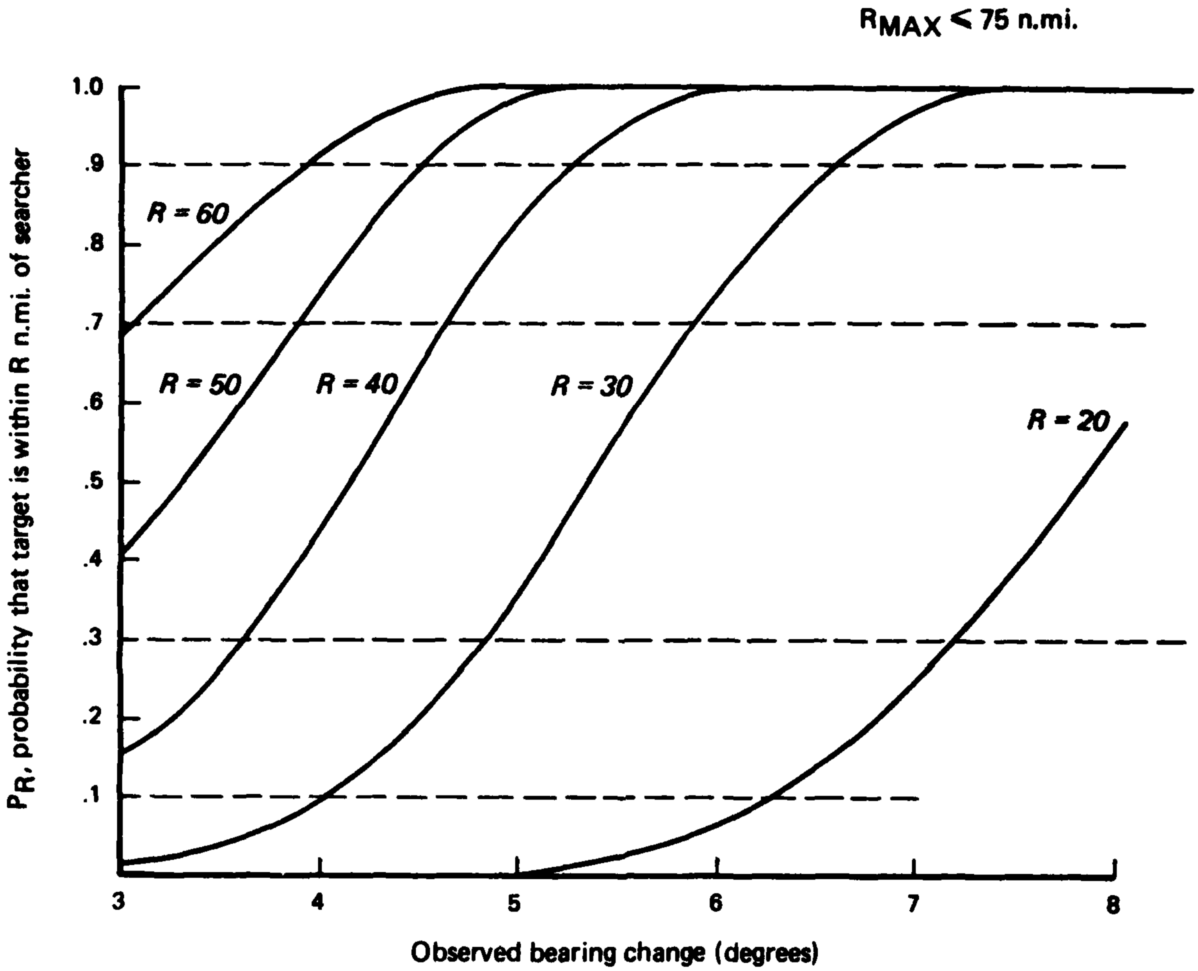

FIG. 3: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 45 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

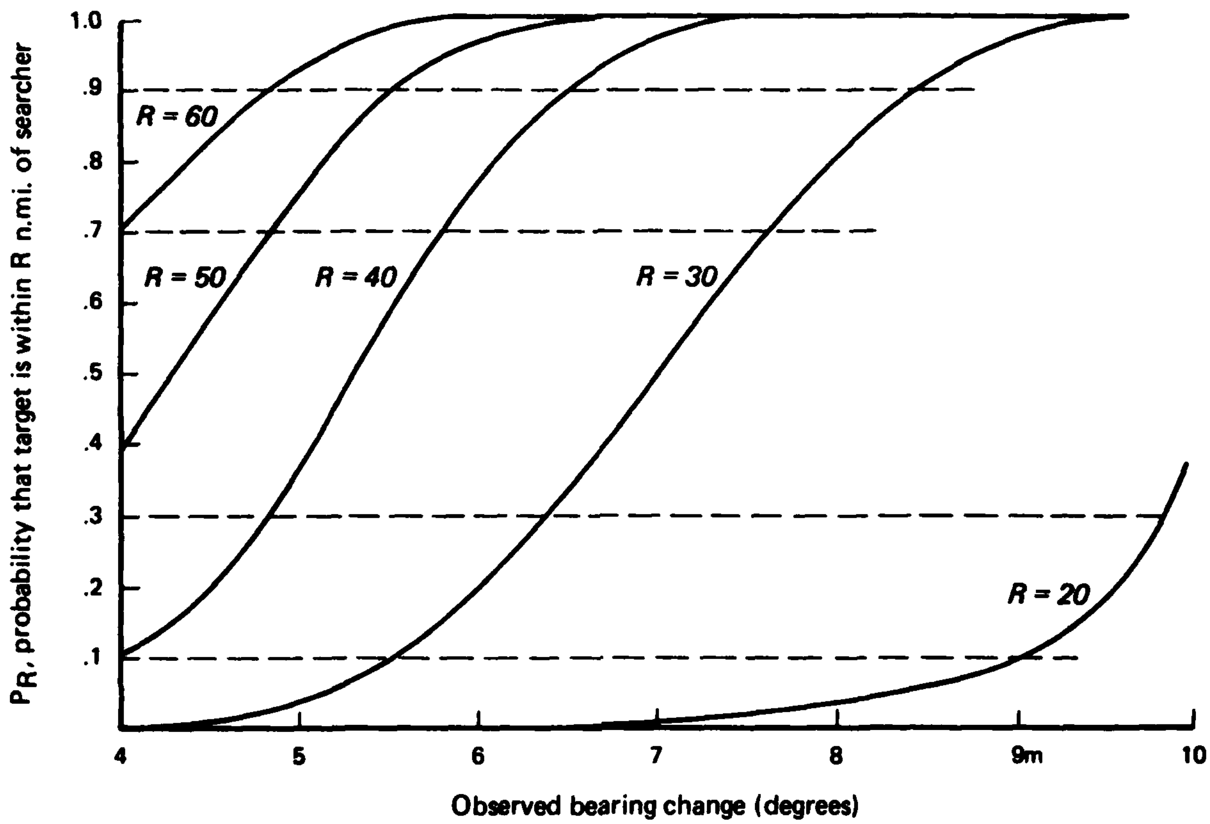

FIG. 4: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 60 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

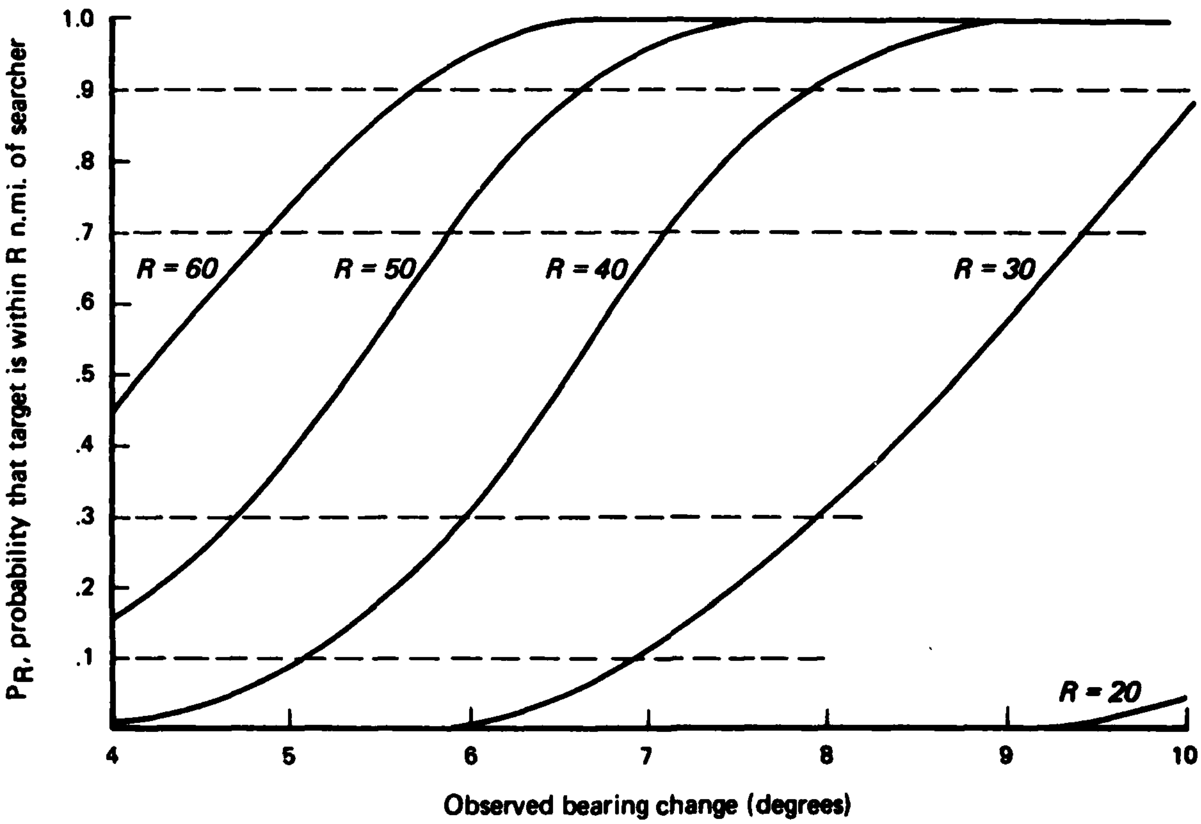

FIG. 5: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 75 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

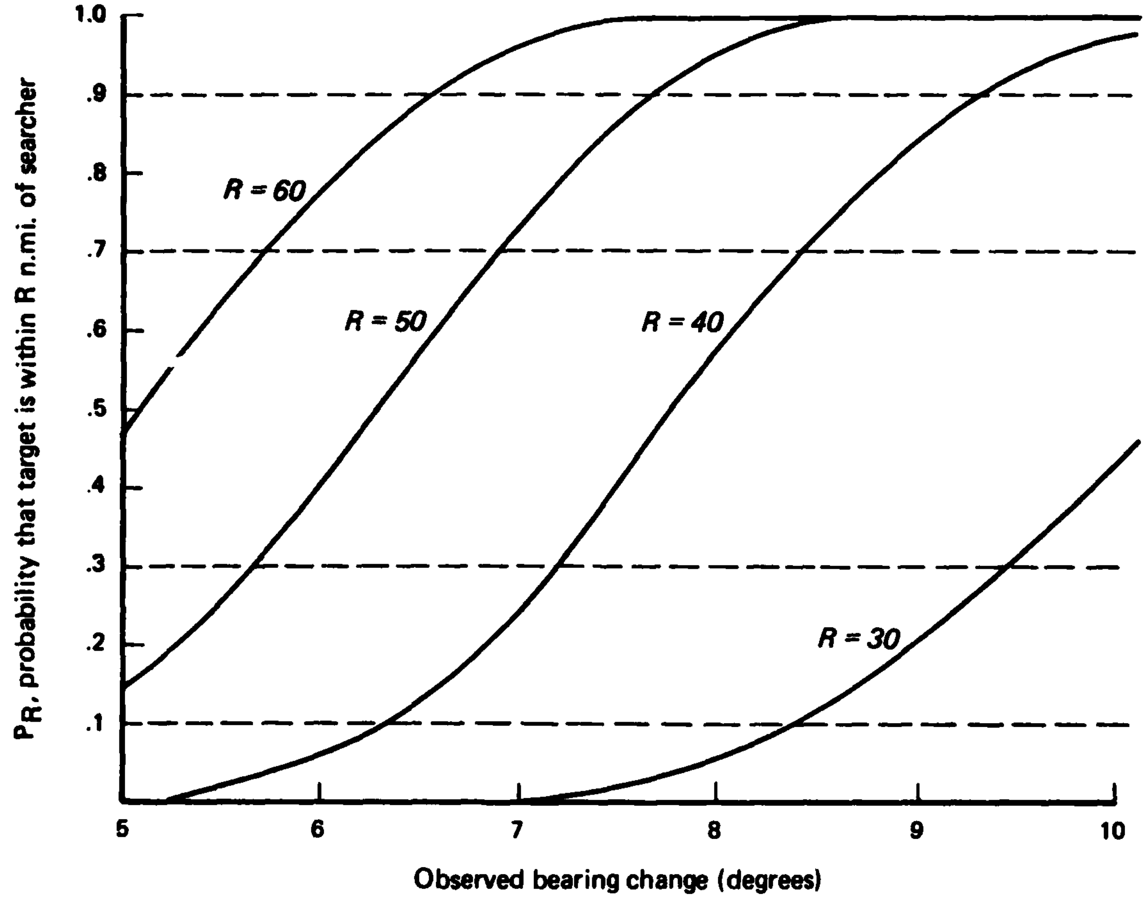

FIG. 6: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 90 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

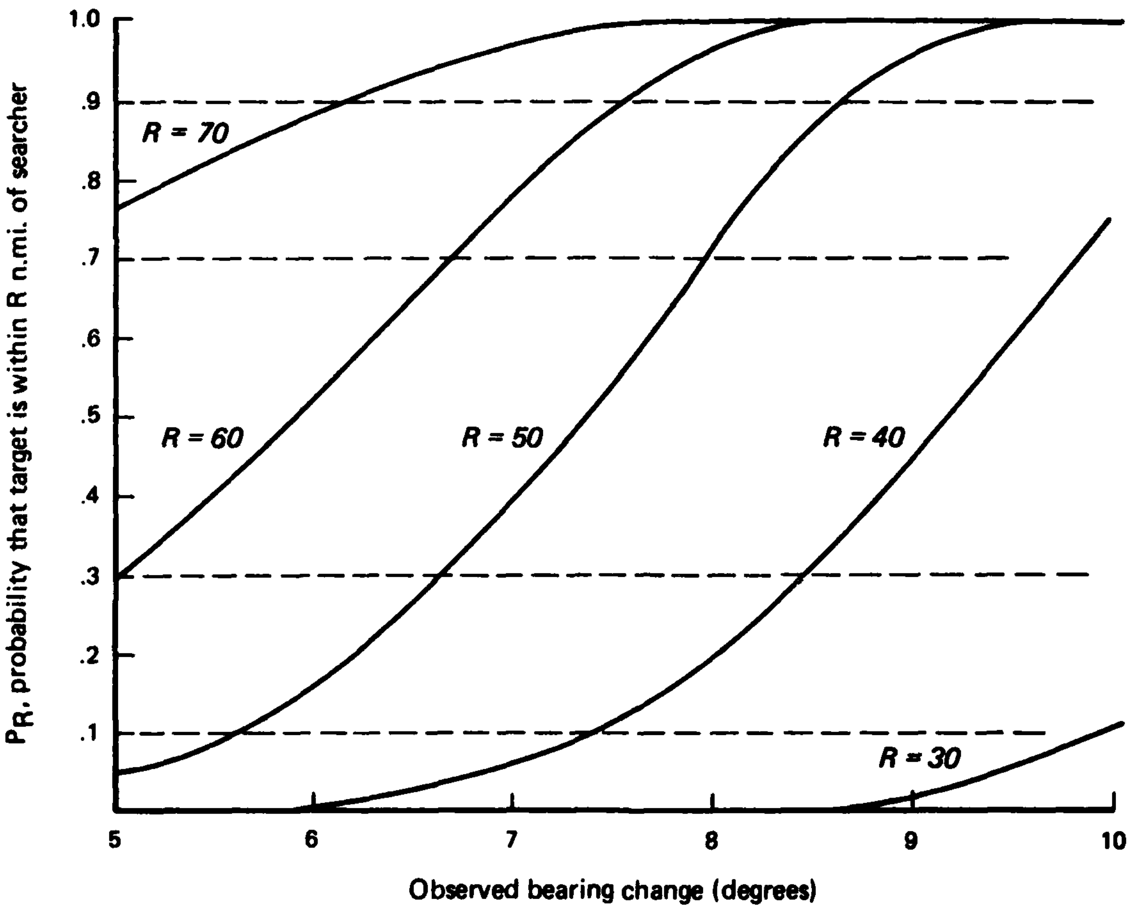

FIG. 7: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 105 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

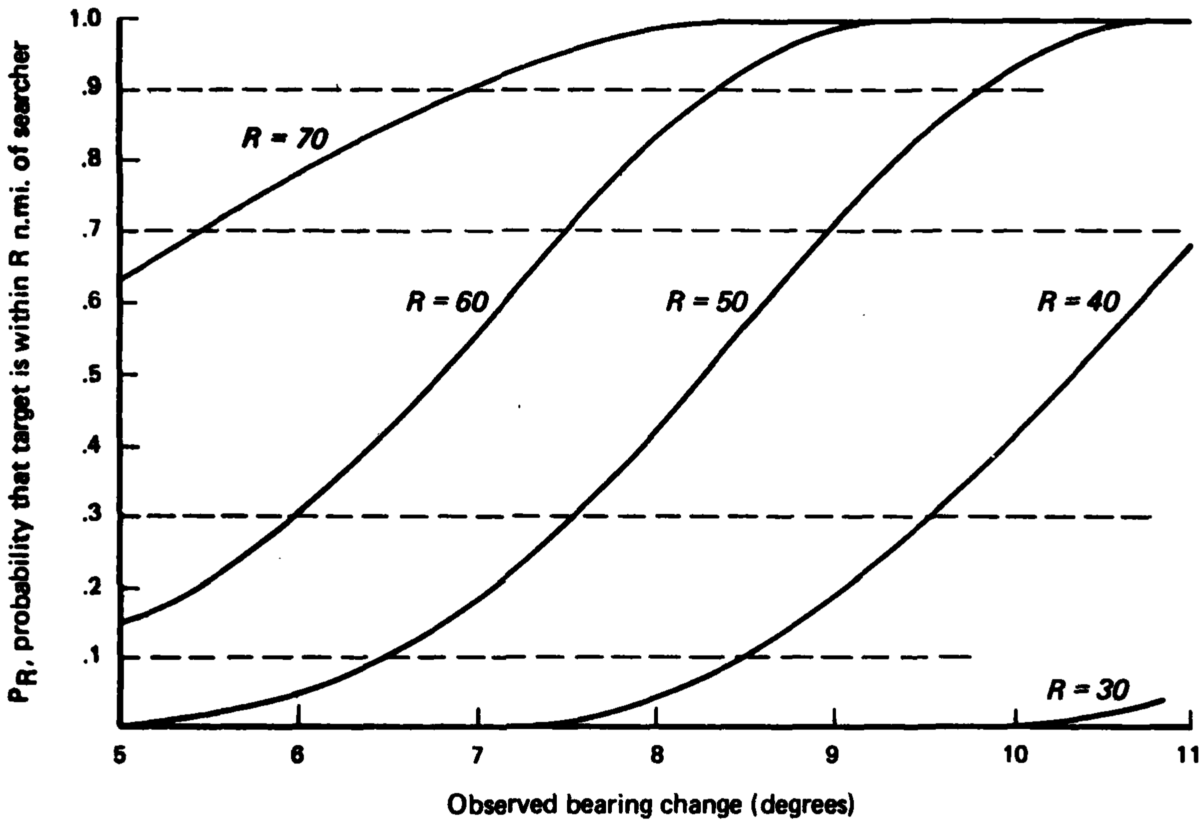

FIG. 8: DISTRIBUTION OF TARGET RANGES FOR OBSERVED BEARING CHANGES WITH V_A = 420 KTS, V_s = 30 KTS, t = 120 SECONDS, AND BEARING ERRORS UNIFORMLY DISTRIBUTED BETWEEN ±1° AND R_{MAX} < 75 n.mi. (1,000 REPLICATIONS PER BEARING CHANGE WITH INITIAL BEARING OF 30°)

3. Compute real bearings according to

$$\phi_0 = \hat{\phi}_0 + \delta_1$$
$$\phi_1 = \hat{\phi}_1 + \eta_1$$

here δ_1 and η_1 are independently identically distributed random variables, with a uniform distribution on $[-\delta°, \delta°]$. Here δ is a specified angle.

4. Compute R from equation (3.1).

5. Recompute statistics for the distribution on R.

6. N is replaced by $N+1$; if the new value is less than the cutoff, return to step 2, otherwise exit.

Simulation Results

It is worthwhile to consider how the random range and random bearing simulations differ. In the random bearing simulation, the conditional distribution function for R, given the observations

$$H(r) = \Pr\{R < r \mid \hat{\phi}_0, \hat{\phi}_1\} \tag{3.3}$$

is computed directly. In the random range simulation, the distribution function on R is initially assmed to be uniform on $[R_{min}, \hat{R}_{max}]$ (where $\hat{R}_{max}$ is either R_{max} from equation (1.2) or an artificially imposed cut-off). Then trials that do not conform to the assumption that $-1 < \phi_0 - \hat{\phi}_0 < 1$ are rejected. The differences in the two procedures should be reflected most in differences in the tails of the range distributions.

Figure 9 shows the results of the two kinds of simulations. It appears that for most ranges, the R simulation underestimates the closeness of the target.

4. ANALYTICAL RESULTS: MEAN AND VARIANCE OF THE RANGE

First consider the derivation of equation (3.1). The geometry is shown in figure 10. Set $S = V_A t$ and $d = V_S t$. Applying the law of sines to the small triangle gives

$$\frac{d}{\sin(\phi_1 - \phi_0)} = \frac{Q}{\sin(\phi_0 - \alpha)} \tag{4.1}$$

For the large triangle, the law of sines gives

$$\frac{S}{\sin(\phi_1 - \phi_0)} = \frac{Q+R}{\sin(\phi_0)} \tag{4.2}$$

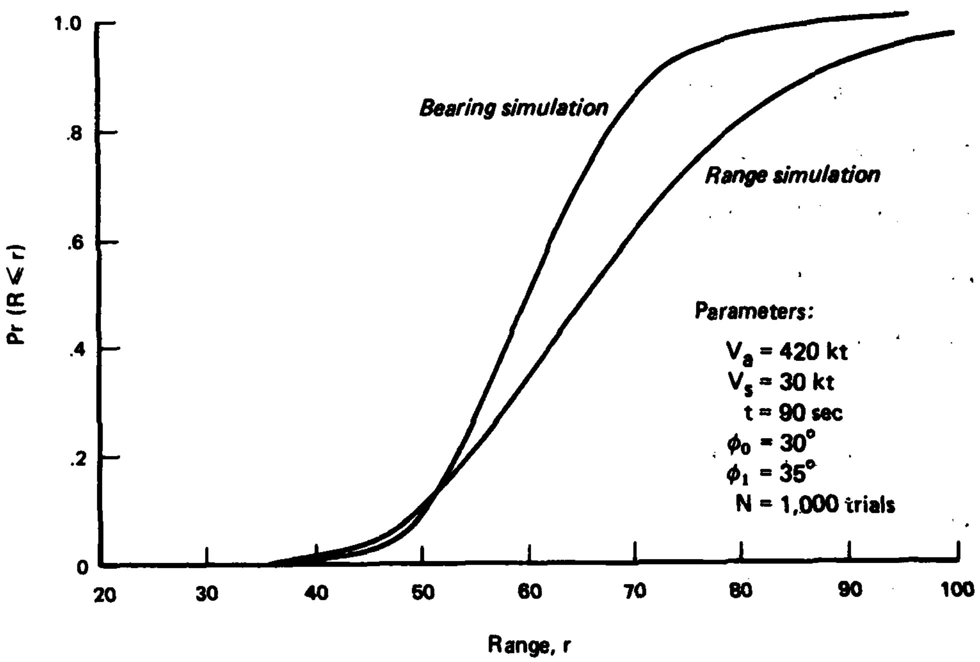

FIG. 9: COMPARISON OF RANGE AND BEARING SIMULATIONS

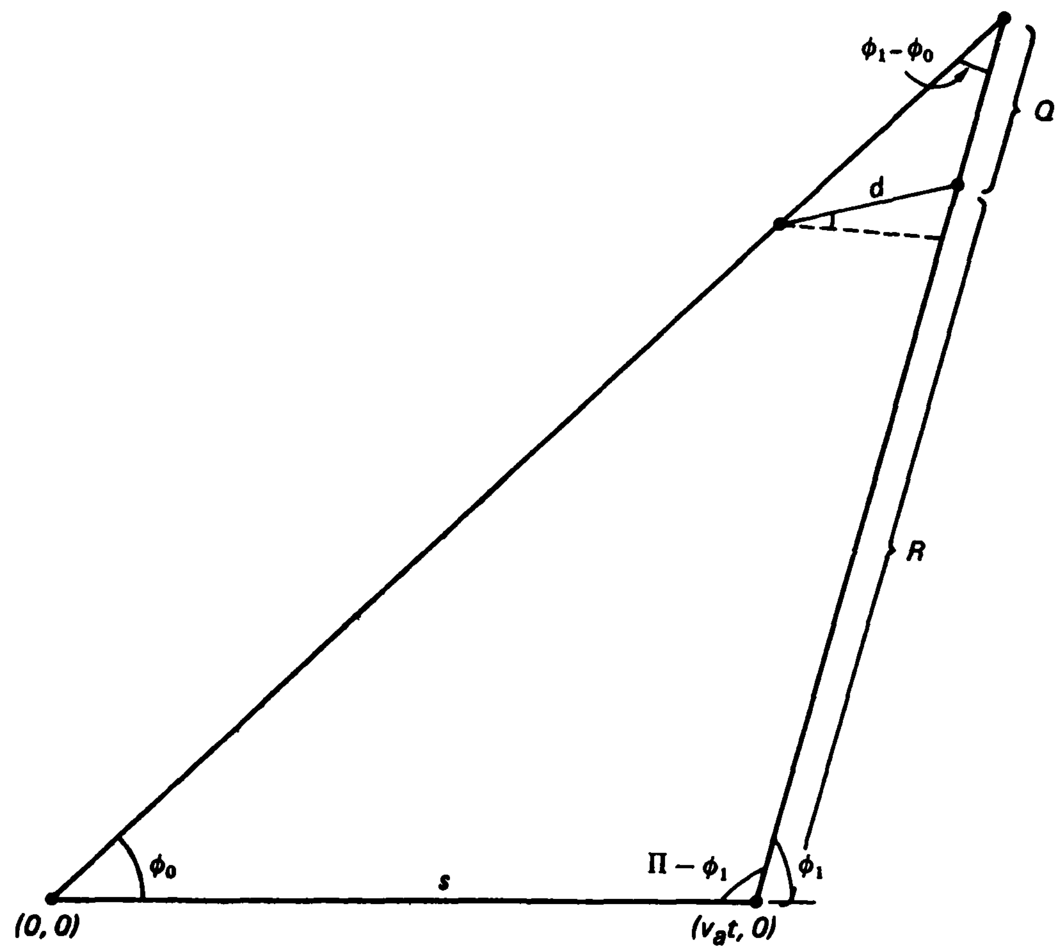

FIG. 10: GEOMETRY USED IN DERIVATION OF EQUATION 3.1.

Solving equations (4.2) and (4.3) for R gives

$$R = \frac{S \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - Q$$

$$= \frac{S \sin(\phi_0)}{\sin(\phi_1 - \phi_0)} - \frac{d \sin(\phi_0 - \alpha)}{\sin(\phi_1 - \phi_0)} \qquad (4.3)$$

Now replace ϕ_0 and ϕ_1 with random variables, X and Y. R is a random variable too. Furthermore, R is a nonlinear function of the random variables X and Y. In order to calculate the mean and variance of R, the following approximation is used. If $f(X,Y)$ is any twice differentiable function, and $\bar{X} = E\{X\}$, $\bar{Y} = E\{Y\}$, then a Taylor expansion gives the approximation

$$E\{f(X,Y)\} \cong E\{f(\bar{X},\bar{Y}) + f_x(\bar{x},\bar{y})\,[X-\bar{X}] + f_y(\bar{X},\bar{Y})\,[Y-\bar{Y}]$$

$$+ \frac{1}{2} f_{xx}(\bar{X},\bar{Y})\,[X-\bar{Y}]^2 + f_{xy}(\bar{X},\bar{Y})\,[X-\bar{X}][Y-\bar{Y}] \qquad (4.4)$$

$$+ \frac{1}{2} f_{yy}(\bar{X},\bar{Y})\,[Y-\bar{Y}]^2\}$$

so that

$$E\{f(X,Y)\} \cong f(\bar{X},\bar{Y}) + \tfrac{1}{2}[f_{xx}(\bar{X},\bar{Y})\ \mathrm{Var}\ X + 2f_{xy}(\bar{X},\bar{Y})\ \mathrm{Cov}(X,Y) + f_{yy}(\bar{X},\bar{Y})\ \mathrm{Var}\ Y] \tag{4.5}$$

In the same way one finds that

$$\begin{aligned} E\{f^2(X,Y)\} \cong f^2(\bar{X},\bar{Y}) &+ [f_x^2(\bar{X},\bar{Y}) + f(\bar{X},\bar{Y})\ f_{xx}(\bar{X},\bar{Y}))]\ \mathrm{Var}\ X \\ &+ [f_y^2\ (\bar{X},\bar{Y}) + f(\bar{X},\bar{Y})\ f_{yy}(\bar{X},\bar{Y})]\ \mathrm{Var}\ Y \\ &+ 2\ [f_x(\bar{X},\bar{Y})\ f_y(\bar{X},\bar{Y}) \\ &+ f(\bar{X},\bar{Y})\ f_{xy}(\bar{X},\bar{Y})]\ \mathrm{Cov}\ (\bar{X},\bar{Y}) \end{aligned} \tag{4.6}$$

In the case of interest here, $f(X,Y)=R=[S\ \sin(X) - d\ \sin(X-\alpha)][\sin(Y-X)]^{-1}$, and assuming $\mathrm{Cov}(X,Y)=0$, the derivatives of interest are

$$R_y = -[S \sin(x) - d \sin(x-\alpha)] [\sin(y-x)]^{-2} \cos(y-x)$$

$$R_{yy} = -[S \sin(x)-d \sin(x-\alpha)] \{-2[\sin (y-x)]^{-3} \cos^2(y-x)$$

$$- [\sin (y-x)]^{-1}\}$$

$$(4.7)$$

$$R_x = [S \cos(x)-d \cos(x-\alpha)] [\sin(y-x)]^{-1}$$

$$+ [S \sin(x) - d \sin(x-\alpha)] [\sin(y-x)]^{-2} \cos (y-x)$$

$$R_{xx} = 2[S \cos(x) - d \cos(x-\alpha)] [\sin(y-x)]^{-2} \cos(y-x)$$

$$+ 2[S \sin(x) - d \sin(x-\alpha)] [\sin(y-x)]^{-3} \cos^2(y-x)$$

The derivatives (4.7) involve $\cos(x-\alpha)$ and $\sin(x-\alpha)$. Assuming the $\alpha \sim U[w_0,w_1]$, table 1 shows the averages of the functions of interest.

TABLE 8

Function	w_0	w_1	Average
$\cos(x-\alpha)$	0	360°	0
$\sin(x-\alpha)$	0	360°	0
$\cos(x-\alpha)$	0	360°	$2 \sin x/\pi$
$\sin(x-\alpha)$	0	360°	$-2 \cos x/\pi$

Note that if α is uniformly distributed on [0, 360°], then the target motion makes essentially no contribution to the results of this level of approximation. Using equations (4.5) - (4.7) and table 8, assuming X and Y are independent, and $X,Y \sim U[-1°, 1°]$ leads to figures 11,12, and 13. In these figures, the mean of R is plotted against $\Delta\phi \equiv \phi_1-\phi_0$ for t = 30, 40, and 50 seconds, with $\alpha \sim U[0,2\pi]$, V_A = 420 kts, and V_S = 30 kts. The error bars indicate one standard deviation.

The considerable size of the error bars suggests that distributional results are very important, since knowledge of the mean and variance is not sufficient in this case to give accurate predictions.

5. ANALYTICAL RESULTS: APPROXIMATE CALCULATION OF THE DISTRIBUTION ON R

Since R is a nonlinear function of X and Y, it is difficult to calculate its distribution exactly. In this section, an approximate method for the calculation of the distribution is given. The starting point is equation (4.3) for R with X and Y replacing ϕ_0 and ϕ_1:

$$R = \frac{S \sin(X)}{\sin(Y-X)} - \frac{d \sin(X-\alpha)}{\sin(Y-X)} \tag{5.1}$$

The approximation used here is the small noise approximation, in which X and Y are replaced by

$$\begin{aligned} X &= \bar{X} + \delta \\ Y &= \bar{Y} + \eta \end{aligned} \tag{5.2}$$

where $\delta \sim U[-\delta_1, \delta_1]$ and $\eta \sim U[-\eta_1, \eta_1]$

are the deviations from the average, and a Taylor expansion of (5.1) is used to write R as a linear function of δ and η. Such a procedure is valid if δ and η are sufficiently small relative to other terms in the expansion. Although used mainly for mathematical convenience, if such an approximation is not valid the entire concept of getting useful information from two noisy bearings may not be valid, since then the noise overrides the deterministic terms.

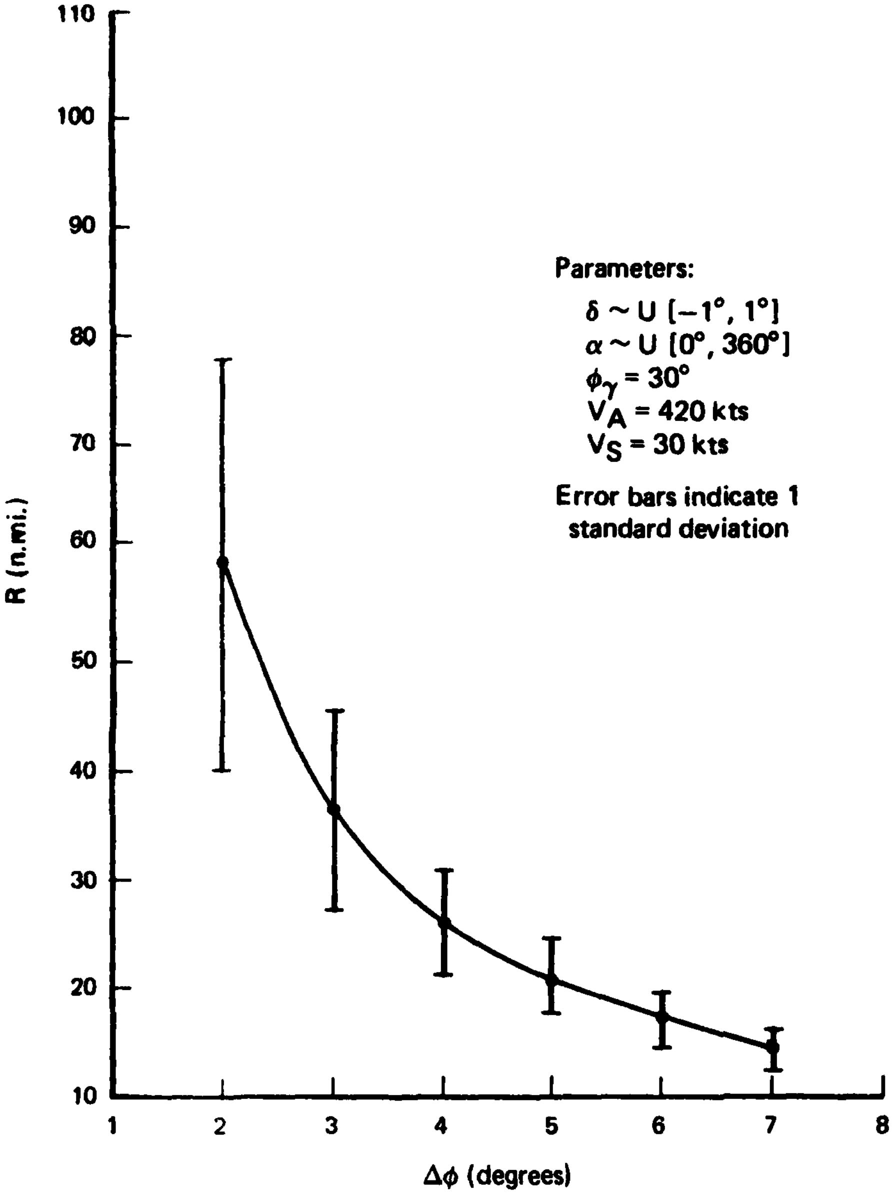

FIG. 11: R vs., $\Delta\phi$ WITH t = 30 SECONDS

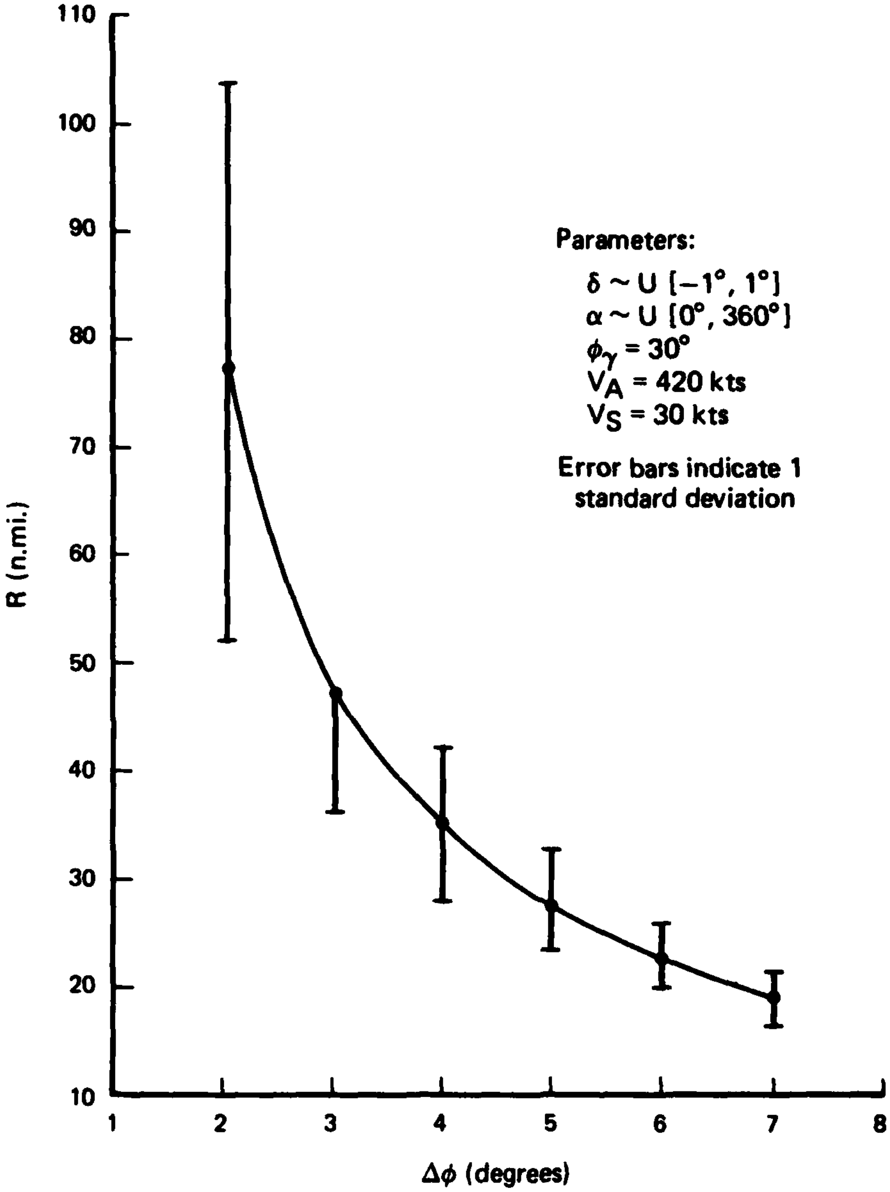

FIG. 12: R vs. $\Delta\phi$ WITH t = 40 SECONDS

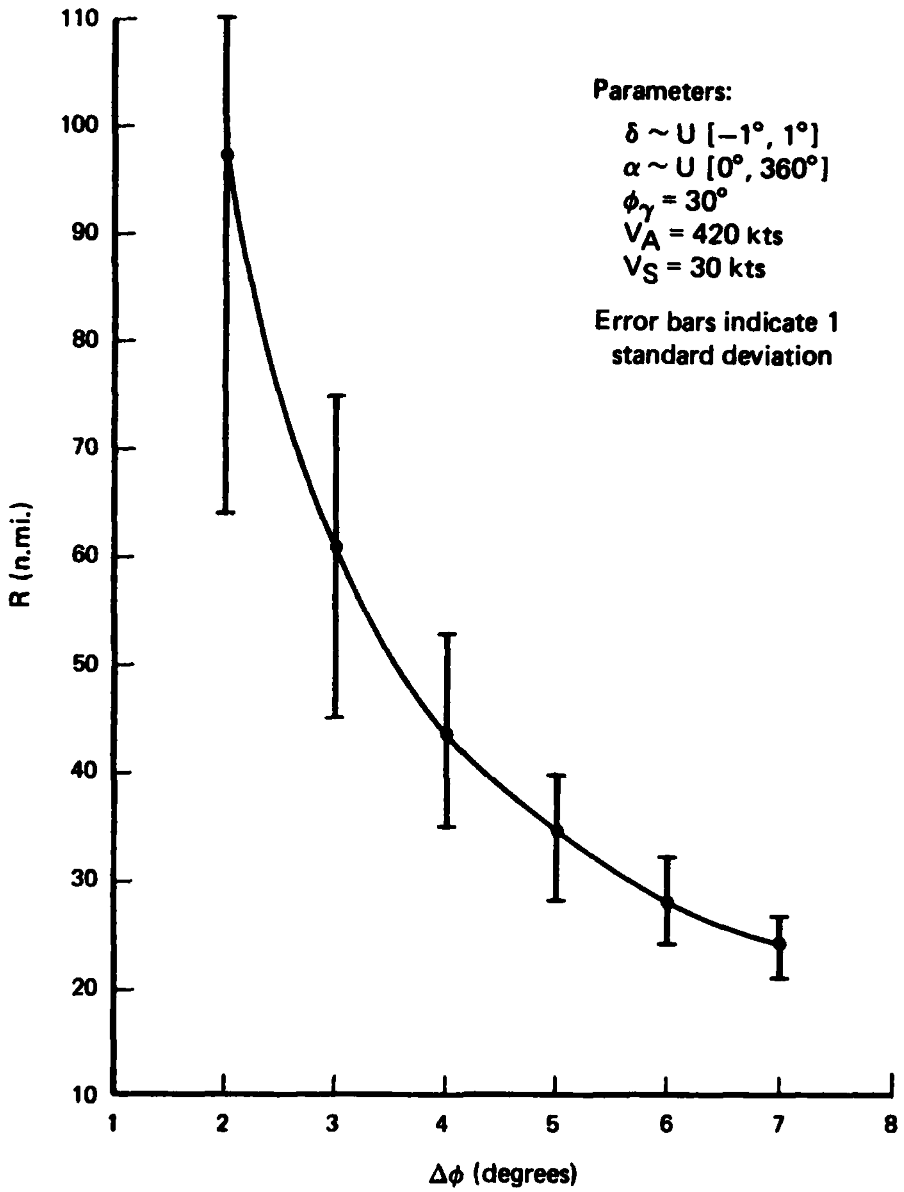

FIG. 13: R vs. $\Delta\phi$ WITH t = 50 SECONDS

In this approximation

$$\sin(X) \cong \sin(\bar{X}) + \delta \cos(\bar{X}) \equiv A_1 + A_2\delta$$

$$\sin(X-\alpha) \cong \sin(\bar{X}-\alpha) + \delta \cos(\bar{X}-\alpha) \equiv C_1 + C_2\delta$$

$$[\sin(Y-X)]^{-1} \cong [\sin(\bar{Y}-\bar{X})]^{-1} - \frac{\cos(\bar{Y}-\bar{X})}{[\sin(\bar{Y}-\bar{X})]^2}(\eta-\delta) \equiv B_1 - B_2(\eta-\delta) \tag{5.3}$$

Using (5.3) in (5.1) gives

$$R = \bar{R} + E\delta + F\eta \tag{5.4}$$

where

$$\bar{R} = B_1(SA_1 - dC_1)$$

$$E = B_1(SA_2 - dC_2) + B_2(SA_1 - dC_1) \tag{5.5}$$

$$F = B_2(dC_1 - SA_1)$$

The distributional quantity of interest $H(r)$ is then

$$H(r) = \Pr\{R < r\} = \Pr\{\bar{R} + E\delta + F\eta < r\} \tag{5.6}$$

It is easy to show that, for the operational values of interest, $E>0$ and $F<0$. In light of the form of R in (5.4), there are maximum and minimum possible values of R given by

$$\begin{aligned} R_{max} &= \bar{R} + E\delta_1 + |F|\eta_1 \\ R_{min} &= \bar{R} - E\delta_1 + F\eta_1 \end{aligned} \tag{5.7}$$

Note that R_{max} and R_{min} still depend upon α through the coefficients in (5.3) and (5.5).

Define new random variables, U and V by

$$U = E\delta \qquad V = |F|\eta \tag{5.8}$$

so that

$$\begin{aligned} U &\sim U[-E\delta_1,\ E\delta_1] \\ U &\sim U[+F\eta_1,\ -F\eta_1] \end{aligned} \tag{5.9}$$

In the sequel it is helpful to set $u_1 = E\delta_1$, $v_1 = |F|\eta_1$. Defining $Z \equiv r-\bar{R}$, equation (5.6) becomes

$$\Pr\{R < r\} = \Pr\{U - V < Z\} \tag{5.10}$$

In order to compute (5.10), refer to figure 14. For a given Z, $\Pr\{U - V < Z\}$ is the fraction of the area of the rectangle above the line $u - v = Z$. There are three cases. Set $Z_1 = u_1 - v_1$ and $Z_2 = v_1 - u_1$; then the three cases are

Case I : $Z > Z_1$

Case II: $Z < Z_2$

Case III: $Z_2 < Z < Z_1$

In Case I

$$\Pr\{U - V < Z\} = 1 - \frac{1}{8u_1v_1}(u_1 + v_1 - Z)^2 \tag{5.11}$$

In Case II,

$$\Pr\{U - V < Z\} = \frac{1}{8u_1v_1}(Z + u_1 + v_1)^2 \tag{5.12}$$

In Case III,

$$\Pr\{U - V < Z\} = 1 - \frac{1}{4u_1v_1}\{2v_1^2 + 2v_1(u_1 - Z - v_1)\} \tag{5.13}$$

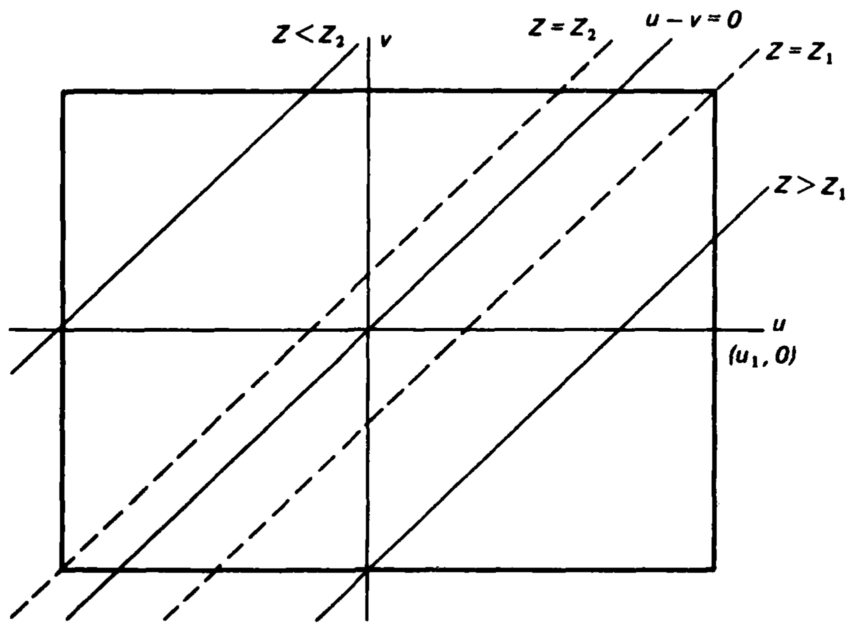

FIG. 14: RECTANGLE USED TO COMPUTE Pr $U - V < Z$

Figure 15 shows a comparison of the theory developed here with the range and bearing simulations for the same parameters as in figure 9. Further comparisons of the theory and bearing simulation, with the noise uniformly distributed on $\pm .5°$ or $\pm .25°$ showed that the agreement between theory and simulations was very good.

Further comparisons of the theory and the random range simulation showed that the predicted minimum values for the ranges agreed closely, but that the predicted maximum values disagreed. As an example, for the parameter values in figure 9, the minimum range by the theory is 34 n.mi. and the minimum range by the range simulation is 35 n.mi. The maximum ranges, however, are 86 n.mi. and 117 n.mi. respectively. The cause of the long tail on the range simulation is not known.

TACTICAL IMPLICATIONS AND USES

One tactical use of the techniques described here is the determination of confidence levels on the range to a target. Referring to figure 16, if the observer wished to know that he was within 60 miles of the target with .9 probability, he would need to observe a bearing after t seconds that was above the $P_{60} = .9$ curve. If the observer wished to know that he was outside 30 miles of the target with .9 probability, he would need to observe a bearing after t seconds that was below the $P_{30} = .1$ curve. Finally, if the observer wished to know that he

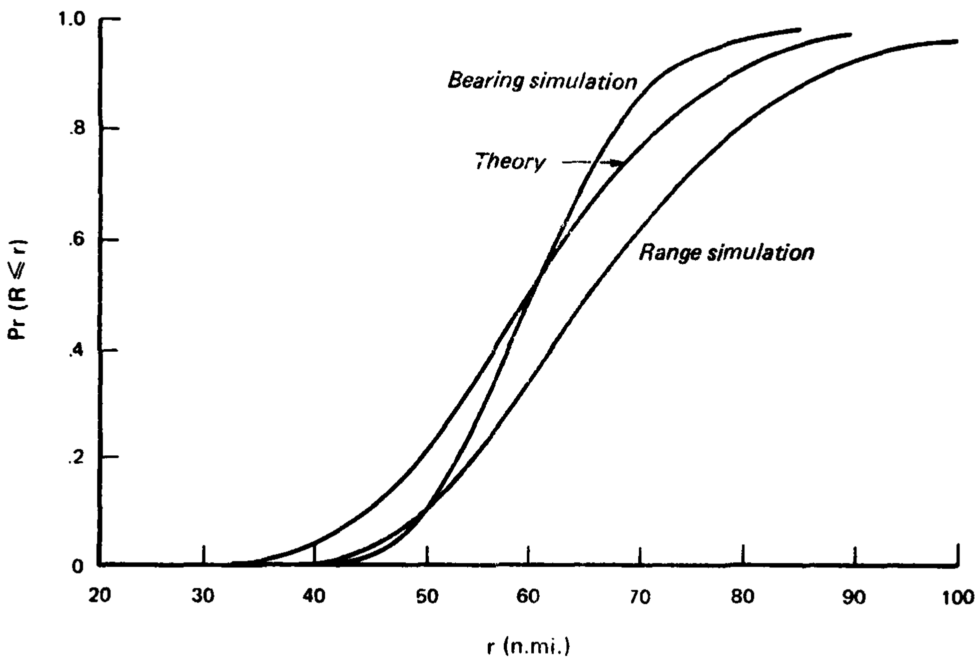

FIG. 15: COMPARISON OF THEORY WITH RANGE AND BEARING SIMULATIONS

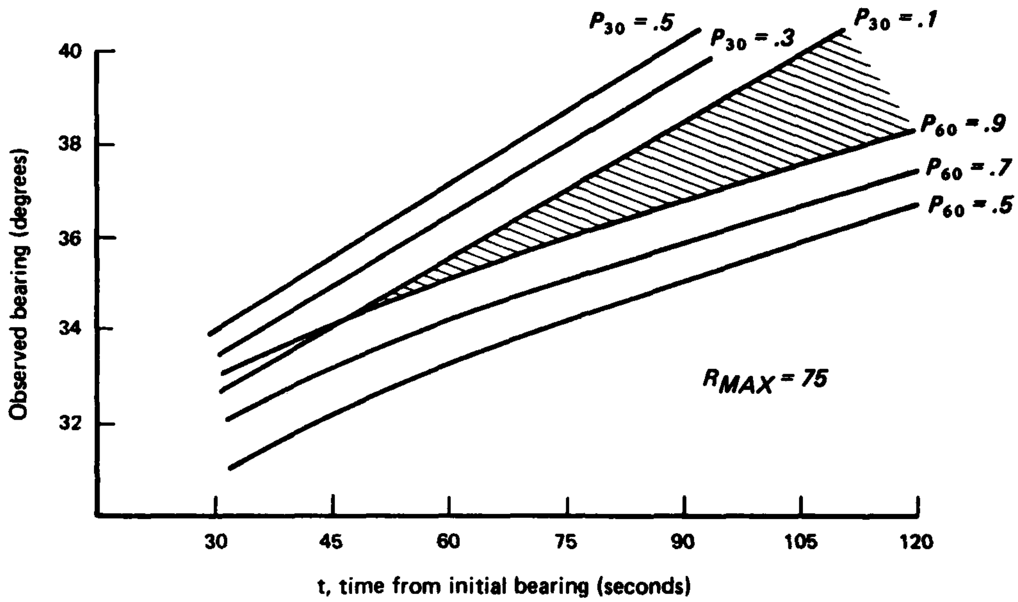

FIG. 16: P_{60} AND P_{30} PROFILES FOR BEARING OBSERVED t SECONDS AFTER THE INITIAL BEARING OF 20 DEGREES

was somewhere between 30 and 60 miles from the target with .9 probability, his bearing observation after t seconds would need to be below the $P_{30} = .1$ curve but above the $P_{60} = .9$ curve. This is the shaded region shown in the figure.

There may be other applications for this sort of information. For any application to be operationally useful it will be necessary to determine through operational testing the actual distribution of bearing errors encountered in the bearing observations.

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